Corruption in Auctions

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Abstract

In the context of an All-Pay Auction where the seller of an indivisible good allocates it to one of many bidders with the intermediation of an auctioneer, we study the effects of two types of corruption on symmetric bidding behaviour and welfare of agents. We find that the effects of corruption on bidding and welfare differ when the corrupt auctioneer colludes with the highest bidder, in comparison to the case where the corrupt auctioneer has a favourite bidder to whom he wishes to allocate the good before observing bids. We give conditions under which bidders become more(less) aggressive than in the benchmark no corruption case for both corruption settings, and also discuss the welfare effects of corruption under these conditions.

1 Motivation and Literature

When studying corruption, a quite rich environment to place it in is that of an auction, as auctions are mechanisms that involve an intermediary (the auctioneer) who is assigned by some seller to allocate a good to some buyer after observing all buyers' bids. In the presence of the intermediary, the mechanism is not immune to corruption. Thus, a substantial part of the procurement literature is built in auction environments. Also, the lobbying procedure has been often modeled as an all-pay auction. The present paper wishes to explore symmetric equilibrium bidding behaviour and its effects on welfare, in the setting of an All-Pay Auction where corruption can occur. We study two different types of corruption schemes: one where the corrupt auctioneer wishes to collude with the winner of the auction, and one where the corrupt auctioneer has a preferred bidder whom he wishes to allocate the good to before observing bids.
On the effects of corruption on the competitiveness of the relevant market, Celentani and Ganuza(2001) study a procurement problem in which the procurement agent is supposed to allocate the realization of a project according to a competitive mechanism that values bids in terms of the proposed price and quality. Potential bidders have private information about their production costs. Since the procurement agent is also in charge of verifying

delivered quality, in exchange for a bribe, he can allow an arbitrary firm to be awarded the realization of the project and to produce a quality level lower than that announced.If the agent decides to be corrupt,he is randomly matched with a firm,and asks for a bribe in exchange for the agreement that the firm will be awarded the project.They find that corruption may well be increasing in competition.

Lengwiler and Wolfstetter(2009) argue that the assumption that the auctioneer would ask for a bribe from the winner of the auction is not the only choice,as the auctioneer might find it profitable to ask for a bribe from the second highest bidder, which would in turn create inefficiencies. Previous work on the possibility of inefficient outcomes was done by Burguet and Che(2004), who study competitive procurement administered by a corrupt agent who is willing to manipulate his evaluation of contract proposals in exchange for bribes. They find that if the agent is corrupt and has large manipulation power, bribery makes it costly for the efcient rm to secure a sure win, so in equilibrium the efcient rm loses the contract with positive probability.

In public procurement processes,the bids are usually multidimensional. Che (1993) proves that a two-dimensional bidding problem,where one dimension is price, and the second is quality can be transformed into a standard one-dimensional problem. Considering three different auctions schemes (first score,second score and second preferred offer), he finds that under the absence of commitment power from the side of the buyer, the only feasible scoring rule is the utility function of the buyer; in this case,all three auction schemes are equally optimal for the buyer.

On the effects of corruption on symmetric bidding behaviour, Arozamena and Weinschelbaum(2009) provide conditions under which in a First Price sealed bid auction and under the presence of a corrupt auctioneer, agents can bid more or less aggressively than in the standard First Price auction.

In what follows, our ultimate goal is to investigate how corruption affects bidders' symmetric bidding behaviour in two different corruption settings; one where the corrupt auctioneer proposes a bribe **after** observing the bids, and a second one where he has a preferred bidder **before** observing the bids. From now on, we will call the former one "Posterior" Corruption, and the later one "Prior" Corruption. In our analysis we will use an All Pay Auction setting, which has been broadly used to study lobbying procedures. Baye, Kovenock and de Vries(1993), under an all pay auction formulation,find that politicians seeking to maximize political rents, are better off excluding the lobbyists with the highest valuations.

2 The Model

Our setting is one where a seller of an indivisible object wishes to sell it to one of N buyers. The mechanism through which the winner is chosen, is a First Price All-Pay Auction, where buyers place one-dimensional price bids; in the setting of the all pay auction, the highest bidding agent wins the object, and all agents pay their respective bids. The seller assigns the auction procedure to an agent of his, henceforth called the auctioneer, which for simplicity, and without loss of generality is assumed to gain a flat wage, which we normalize to 0.
We consider the incomplete information setting, where N risk neutral (pay-off maximizing) players place bids for the indivisible object; each player i's valuation of the object v_i is private information, however it is common knowledge that valuations are iid distributed over $[0, 1]$ with distribution F.We assume that F has full support and positive and continuous density f.

2.1 Benchmark Case: All-Pay Auction without Corruption

As a first step, we discuss the agents' bidding behaviour without corruption. Under no corruption, the highest bidder wins the object,however all bidders have to pay their respective bids; in case of a tie,we assume that one of the tying bidders is randomly assigned the object.Thus,the payoffs for player i are:

$$\pi_i = \left\{ \begin{array}{c} \mathrm{v}_i - b_i \text{ , if } b_i > b_j \ \forall j \neq i \\ \text{-}\mathrm{b}_i \text{ , if } b_i < b_j \text{ for some } j \neq i \\ \frac{v_i}{\#\{m:b_m=b_i\}} - b_i \text{ , if } b_i = b_m > b_j \forall i, m \neq j \text{ and } \forall i \neq m \end{array} \right\}$$

We focus only in the symmetric equilibrium and strategies β increasing in the valuation. As standardly used in auction theory: $\beta(0) = (0)$. Assume all players but player i bid given the increasing strategy $\beta()$. Then, bidder i wins by placing bid b, if $b \geq \beta(y) \forall j \neq i \Rightarrow y \leq \beta^{-1}(b)$, where y is the highest of the remaining $N-1$ valuations, i.e. the highest order statistic of the $N-1$ remaining values. Thus, i maximizes his expected payoff (we drop subscript i for simplicity) :

$$\max \pi_b = vF^{(N-1)}(\beta^{-1}(b)) - b$$

In a symmetric equilibrium $b = \beta(v)$, and thus:

$$\frac{v(N-1)F(v)^{(N-2)}f(v)}{\beta'(v)} = 1 \Rightarrow v(N-1)F(v)^{(N-2)}f(v) = \beta'(v)$$

Keeping in mind that $\beta(0) = 0$, because if positive this would imply losses in the case of a win, the increasing symmetric bidding function is given by:

$$\beta(v) = vF^{(N-1)}(v) - \int_0^v F^{(N-1)}(y)\,dy$$

3 The timing of corruption

After setting up the benchmark case, we now look at two different types of corrupt auctioneers. First, we will consider the situation where the corrupt auctioneer **after** observing the bids, approaches the highest bidder and proposes a bribe scheme,to be described later. Second, we will study an environment where the corrupt auctioneer approaches a favoured bidder **before** observing the bids. It is expected that these two different settings will produce different bidding behaviour, and we wish to compare the effect of posterior and prior corruption on welfare.

3.1 "Posterior" Corruption

After observing bids, the auctioneer approaches the highest bidder, and proposes to lower his bid to the second highest bid. In return, he asks for a bribe, proportional to the difference of the highest and second highest bid; Thus, if v is the highest valuation,and y the second highest, the bribe for the auctioneer is: $\alpha(\beta(v) - \beta(y))$, where α is exogenous.
The expected payoff of a bidder i when all other bidders are playing given a symmetric increasing bidding function $\beta()$ then is:

$$\mathrm{E}\,\pi = [(v - \mathrm{E}\,\beta(y)|y < v) - \alpha(b - \mathrm{E}\,\beta(y)|y < v)]G(\beta^{-1}(b)) - b[1 - G(\beta^{-1}(b))]$$

,where we have substituted for simplicity $F^{N-1}() = G()$. When the bidder does not have the highest bid he loses and pays his own bid, when he wins, he pays the second highest bid, plus proportion α of the difference of the two bids. We want to pin down the increasing symmetric bidding function β. Instead of maximizing expected profits, we will compute $\beta()$ using the property of Revenue Equivalence.[1]Since bidders are risk neutral and symmetric, and face no budget constraints, Revenue equivalence holds, and thus we know that:

$$\mathrm{E}[\beta(y) + \alpha(\beta(v) - \mathrm{E}\,\beta(y)|y < v)]G(v) + \beta(v)[1 - G(v)] = \int_0^v yg(y)\,dy.$$

[1]See for example Krishna(2002)

, where the LHS is the expected payment of bidder i in equilibrium. Because of symmetricity, we have substituted $b = \beta(v)$.
The expectation of $\beta(y)$ conditional on $y < v$ is given by:

$$\frac{1}{G(v)} \int_0^v \beta(y) g(y)\, dy.$$

Substituting and taking derivatives with respect to v, we end up with the following differential equation[2]:

$$\beta'(v) = \frac{v g(v)}{1 - (1-\alpha)G(v)}$$

Let $\frac{g(v)}{1-(1-\alpha)G(v)} \equiv \phi'(v)$. Then,integrating both sides and then integrating by parts, we get the increasing symmetric strategy equilibria of the posterior corruption auction:

$$\beta^{PostC}(v) = v\frac{\log[1-(1-\alpha)G(v)]}{(1-\alpha)} - \int_0^v \frac{\log[1-(1-\alpha)G(y)]}{(1-\alpha)}\, dy.$$

Proposition 1 The limit of $\beta^{PostC}(v)$ as $\alpha -> 1$ is equal to $\beta(v)$, the increasing symmetric equilibrium bidding function in the benchmark no corruption case. Also, $\beta^{PostC}(v)$ at $\alpha = 0$ is equal to $\beta^{SP}(v)$, the increasing symmetric equilibrium bidding function in a Second Price All Pay auction, where the highest bidder wins the object and pays the second highest bid, and all other bidders forfeit their bids.
The proof for the first part of Proposition 1 is straightforward. Proof for the second part is provided in Appendix A. Notice that since $\alpha \leq 1$, and $log(x)$ is an increasing function of x, then $\frac{\partial \beta^{PostC}(v)}{\partial \alpha} < 0 \rightarrow \beta^{PostC}(v)$ decreasing in α. Thus, the smaller the share of the surplus $\beta(v) - \beta(y)$ the bidder has to pay to the corrupt auctioneer, the more aggressive bidders become.
Proposition2 There does not exist an increasing symmetric equilibrium bidding function, when the highest losing bidder is approached by the corrupt auctioneer, and subsequently when any of the losing bidders is approached by the auctioneer.
Although we do not provide a formal proof for Proposition 2 yet, the intuition behind it is the following: if the corrupt auctioneer approaches the second highest bidder, he offers to match his bid with the highest bid, and thus secure him a win, in exchange of a bribe, proportional to the difference

[2] $\beta(0) = 0$

between the highest and the second highest bid. The bribing scheme is accepted, only if the valuation of the second highest bidder is bigger(or equal) to what the second highest bidder has to pay. Then for some v's and some α's, it must be the case that the second highest bidder rejects the offer of the corrupt auctioneer, and the object is allocated to the highest bidder. The maximization problem of the agent is therefore(we consider the two bidder case with fixed bribe B for simplicity,and without loss of generality):

$$max\pi_b = [(v-b)F(b+B) - b[1-F(b+B)]F(\beta^{-1}(b))+$$

$$[(v - \tfrac{1}{1-F(v)} \int_v^1 \beta(y)f(y)\,dy\text{-B})\text{F}(\beta^{-1}(v-B)) - b[1-F(\beta^{-1}(v-B))]]$$

$$[1-F(\beta^{-1}(b)]$$

Then, players wish to have the highest bid under some conditions, and have the second highest bid under some other conditions. Thus, an increasing symmetric equilibrium bidding function does not exist[3].

3.2 "Prior" Corruption

Bidders know that before observing bids, with probability λ the auctioneer has a "favourite" bidder,to whom he is willing to give the object. If the "favourite" bidder happens to be the highest bidder, the auctioneer proposes to match his bid to the second highest bid, and they share the surplus $\beta(v) - \beta(v_{corrupt})$,where v is the highest valuation[4], in the fashion described before. If the favoured bidder is not the highest bidder,the auctioneer proposes to increase his bid up to the highest bid,and then the surplus $v_{corrupt} - \beta_v$ is divided as before.This is accepted only if $v_{corrupt} \geq \beta(v) + \alpha(\beta(v) - \beta(v_{corrupt}))$. If the proposal is rejected,the highest bidder gets the object,and pays his own bid. We consider again the increasing symmetric equilibrium bidding behaviour of the non-corrupt bidders, and for simplicity we set $\lambda = 1$.
The non-corrupt bidder then knows that he faces $N-2$ non-corrupt bidders, and one bidder that is corrupt. Then, his bid is competing against $(N-2)$ non-corrupt bids, and also against the valuation of the corrupt bidder. Thus, the maximization problem of the non-corrupt bidder is:

$$\max \pi_b = vF^{(N-2)}(\beta^{-1}(b))F(b) - b$$

The resulting differential equation, after imposing symmetry, is:

$$\beta'(v) = \frac{vg(v)F(\beta(v)}{1 - vG(v)f(\beta(v))}$$

[3]Currently working on the formal proof of the proposition
[4]Keeping in mind that we are looking at symmetric equilibria

, where for simplicity we have set $F^{(N-2)}(\beta^{-1}(b)) \equiv G((\beta^{-1}(b))$.
Although we cannot guarrantee that the symmetric increasing equilibrium is unique(in fact there might be more than one increasing symmetric equilibrium bidding functions), in Appendix B we prove existence and optimality. We wish to study how the existence of a corrupt bidder affects the symmetric bidding behaviour of non-corrupt bidders, in comparison to the benchmark case without corruption; our findings are summarized in the following proposition.

Proposition 3 If $\frac{F(\beta^c(v))}{F(v)}\frac{1}{\beta^{c\prime}(v)} > \frac{f(\beta^c(v))}{f(v)}$ then bidders bid uniformly less aggressively than in the benchmark no corruption case. If $\frac{F(\beta^c(v))}{F(v)}\frac{1}{\beta^{c\prime}(v)} > \frac{f(\beta^c(v))}{f(v)}$ and in addition $\frac{f(\beta^c(v))}{f(v)} > 1$, then bidders bid uniformly more aggressively than in the benchark no corruption case(analogous condition to Arozamena and Weinschelbaum(2009)).

The proof of Proposition 3 is in Appendix C[5]. The intuition about this result, lies in the fact that non-corrupt bidders are driven by two different forces. On one hand, they know that there are cases where they would have won in the benchmark no corruption case, but now lose for sure; this effect drives them to bid less aggressively. However, they also know that if the valuation of the corrupt bidder is not high enough given their own bid, they can win. This effect drives bidders to bid more aggressively. The difference with respect to the First Price Auction, is that in our setting bidders always forfeit their bids, and this leads to less aggressiveness than in the First Price auction.

4 Welfare

In the Posterior Corruption setting, it is straight forward that the corrupt auctioneer is better off than in the benchmark case. The losing bidders are worse off for $\alpha < 1$, since, by Proposition 1, they become more aggressive than in the benchmark case, and thus forfeit increased bids. The seller is worse off, since not only he receives the second highest bid instead of the highest one as the payment of the winner, but also he receives lower bids from all other bidders, since they bid less aggressively than in the benchmark case. Finally, the winning bidder is worse off if the second highest bid plus the bribe exceeds the bid he would place in the benchmark no corruption case; he is better off otherwise.

As in the Posterior Corruption case, also the the Prior Corruption setting, it is straight forward that the auctioneer is better off in the presence of corruption. The losing bidders are worse off when the condition for more aggressiveness holds, and further more, if one of the losing bidders is the

[5]Having pinned down the conditions for more(less) aggressiveness, in the next version of this paper we will try to translate the conditions in terms of the curvature of F(V)

highest bidder, he is more worse off than the rest honest bidders. However, all losing bidders are better off when the condition for less aggressiveness holds, except of course for the highest bidder that lost to the corrupt one, if this is the case. The corrupt bidder is better off if his valuation is high enough to secure his win; he is also better off in case that his valuation is not high enough to secure his win but the condition for less aggressiveness holds. However, he is worse off when his valuation is low and loses the object, and the condition for more aggressiveness holds. The most interesting welfare effect of Prior Corruption, is that there exists a case where even the seller is better off; it is the case where the valuation of the corrupt chosen bidder is not high enough, thus leading to the allocation of the object to the highest bidder, and the condition for more aggressiveness holds. In all other cases, the seller is worse off in the presence of corruption.

5 Conclusions and Future Work

Throughout the present paper we tried to explore symmetric equilibrium bidding behaviour and its effects on welfare, in the setting of an All-Pay Auction where corruption can occur. We considered two different types of corruption schemes: one where the corrupt auctioneer wishes to collude with the winner of the auction, and one where the corrupt auctioneer has a preferred bidder, whom he wishes to allocate the good to before observing bids. We found interesting differences on the aggressiveness of bids between the two corruption settings; in addition we were able to compare players' welfare when corruption does not exist, and in the two corruption environments.
In the next steps for the evolution of the paper, we wish to focus on two issues. The first one is to compare the bidding behaviour and welfare effects of corruption between the two corruption settings. The second one is to answer, how the introduction of probabilities of getting caught being corrupt by the seller and subsequent punishments, would alter our main findings.

Appendix A

We have to find the increasing symmetric equilibrium bidding function for the Second Price All Pay auction, where the highest bidder wins the object and pays the second highest bid, and all other bidders pay their respective bids. Revenue Equivalence implies: $[1-G(v)]\beta(v)+\int_0^v \beta(y)g(y)\,dy=\int_0^v yg(y)\,dy$

Differentiating with respect to v, we get:

$$\beta'(v) = \frac{vg(v)}{1-G(v)} \Rightarrow \beta(v) = \int_0^v \frac{yg(y)}{1-G(y)}\,dy$$

Using the same trick as before, we get:

$$\beta^{SP}(v) = v[-log(1-G(v))] - \int_0^v -log(1-G(y))\,dy$$

, and thus: $\beta^{PostC}(v, \alpha = 0) = \beta^{SP}(v)$

Appendix B

To prove existence we follow Li and Tan(2000) .We are looking for increasing $\beta(v) \in [0,1], \beta(v) \leq v$, with the initial condition $\beta(0) = 0$. Thus, our initial domain is: $\Delta = \{(\beta(v), v) \in [0,1]^2 : \beta(v) \leq v\}$. Our differential equation is:

$$\beta'(v) = \frac{vg(v)F(\beta(v))}{1 - vG(v)f(\beta(v))}$$

The numerator can be equal to 0 within our domain, specifically for $v = 0$, thus, in order to be able to use Existence Theorems we need to exclude from our domain $v = 0$. Define our new domain: $\Delta' = \{(\beta(v), v) \in (0,1]^2 : \beta(v) \leq v\}$. Here, we can apply the standard Existence Theorems[6]. Thus, $\forall (v_0, b_0) \in \Delta'$, with initial condition $b(v_0) = b_0$, there exists a unique solution $\beta(v)$ in the neighbourhood of v_0, and it is continuous in the initial value v_0. Furthermore, notice that $\beta'(v) > 0$ for all $v \in \Delta'$[7].
Now we can prove that for bidder i, if all other non-corrupt bidders bid according to an increasing bidding function $\beta(v_{-i})$, it is optimal to bid $b_i = \beta(v_i)$.
Remember that the expected payoff of the bidder is:

$$\pi_b = vF^{(N-2)}(\beta^{-1}(b))F(b) - b$$

Then:

$$\pi_b' = -1 + v[g(\beta^{-1}(b))\frac{1}{\beta'(\beta^{-1}(b))}F(b) + G(\beta^{-1}(b))f(b)]\textbf{(a)}$$

Also recall that the First Order Condition under symmetry is:

$$1 = v[g(v)\frac{1}{\beta'(v)}F(\beta(v)) + G(v)f(\beta(v))]\textbf{(b)}$$

Plugging **(b)** into (**(a)** yields:

$$\pi_b' = v[g(\beta^{-1}(b))\frac{1}{\beta'(\beta^{-1}(b))}F(b) + G(\beta^{-1}(b))f(b) - g(v)\frac{1}{\beta'(v)}F(\beta(v)) + G(v)f(\beta(v))]$$

[6] See for example Chicone(1999)

[7] In order to conclude the proof, we have to also apply the theorem of complete solutions, to expand our result to the left and to the right along the v axis.

Since all the above functions are increasing, when $\beta^{-1}(b) < v \Rightarrow \pi_b' < 0$, and when $\beta^{-1}(b) > v \Rightarrow \pi_b' > 0$. Thus, it is optimal for bidder i to chose b such that $\beta^{-1}(b) = v$, as there $\pi_b' = 0$.

Appendix C

We want to find conditions under which the bidders in the benchmark no corruption case bid uniformly more aggressively than bidders in our prior corruption all pay auction setting. Then, analogous conditions of uniformly less aggressiveness are straight forward.

In order to prove that under some conditions, $\beta(v) < \beta^c(v)$, we need to show that under these conditions: a) if there exists $\hat{v}$ such that $\beta(\hat{v}) = \beta^c(\hat{v})$, then $\beta'(\hat{v}) < \beta'^c(\hat{v})$, and b) that there does not exist such $\hat{v} \in (0, 1]$.We start by proving the first argument.

As we have seen before, the differential equation in the benchmark no corruption case is:

$$\beta'(v) = v(N-1)F^{N-2}(v)f(v)$$

In our prior corruption case, the analogous differential equation is:

$$\beta'^c(v) = \frac{v(N-2)F^{N-3}(v)f(v)F(b)}{1 - vF^{N-2}(v)f(b)}$$

Now suppose that there exists $\hat{v}$ such that $\beta(\hat{v}) = \beta^c(\hat{v})$. Solving the above differential equations for $\hat{v}$, and then equating them, we get:

$$\frac{1}{\beta'(\hat{v})} = \frac{N-2}{N-1}\frac{F(\beta^c(\hat{v}))}{F(\hat{v})}\frac{1}{\beta^{c\prime}(\hat{v})} + \frac{1}{(N-1)}\frac{f(\beta^c(\hat{v}))}{f(\hat{v})}$$

Notice that for $\frac{f(\beta^c(\hat{v}))}{f(\hat{v})} > \frac{F(\beta^c(\hat{v}))}{F(\hat{v})}\frac{1}{\beta^{c\prime}(\hat{v})}$, $\beta'(\hat{v}) < \beta'^c(\hat{v})$, since $\frac{F(\hat{v})}{F(\beta^c(v))} > 1$. For the proof of the second argument, see Arozamena and Weinschelbaum(2009). They prove by contradiction, that under the condition $\frac{f(\beta^c(\hat{v}))}{f(\hat{v})} < \frac{F(\beta^c(\hat{v}))}{F(\hat{v})}\frac{1}{\beta^{c\prime}(\hat{v})}$, $\beta(v) < \beta^c(v) \forall v \in (0, 1]$.

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